

Once upon a time, there was a town that had no roads, only stepping stones between houses. The mayor decided to pave the stepping stones so that people could begin to drive their cars and ride their bikes. But she only had a limited amount of money to spend on the new roads. She held a contest to see who could come up with the best and cheapest plan. There were two conditions:

- 1. Enough paths must be paved so that each person can travel to everyone else's house without stepping in mud.
- 2. The paving must cost as little as possible.

Each stepping stone costs \$100 to pave. Find the best route that connects all the houses but uses as little money as possible.





New York City Bridges & Tunnels Problem

The map below shows the four land masses and thirteen bridges/tunnels comprising the greater New York City area. Draw a network of vertices and arcs which represents the map. Determine the status (even or odd) of each vertex of the network. Is the network traversable, that is, can every bridge/tunnel be crossed exactly once in a continuous route? If such a route exists, is there any restriction as to where it can begin or end? Explain.



Coloring Maps



by Terry McCabe

Coloring maps can be fun! You might think that the bigger your box of crayons, the better prepared you are to color a map. But suppose you want to color a map using the fewest colors possible so that no two countries with a common border are colored the same. We also assume that each country has at least one border with another country and there are no islands of countries. What is the smallest number of colors guaranteed to color such a map? This is a famous old problem.

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Let's start by drawing a map with three countries (below) that can be colored with just 2 colors. Take another piece of paper and try drawing maps with 4 and 5 countries that require only 2 colors. Do you see a pattern in what a map must be like to require only 2 colors?



Your countries can be rectangular, but if they are, the map may not look realistic.

Now let's draw some maps that require exactly three colors. Draw such maps with 3, 4, 5, 6, 7 and 8 countries respectively. Is there some common feature that each of the maps in this group must have?



For many years, mathematicians believed that any map could be colored using only 4 colors. Proving this was called the "Four Color Problem". Draw some maps that require 4 colors. What is the fewest number of countries a map requiring 4 colors can have?

The answer to the last question is 4 countries. Did you draw such a map? If not, try to draw one now.



In order to see why it was believed that 4 colors was enough, try drawing some maps that require 5 colors. It is a very interesting activity. But please do not spend more than a week or more than a pack of paper on this task.

A group of mathematicians showed in the 1980's that 4 colors is always enough! Using computers to check thousands of possibilities, they showed that no map, no matter how complicated, requires more than four colors. This solved the famous "Four Color Problem" which had baffled mapmakers for centuries.